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A BOOLEAN APPROACH TO SEMANTICS

by

Edward L. Keenan

0. INTRODUCTION

By a boolean approach to model theoretic semantics I intend one in which for each model M and each category C of expression in the language, the set of possible denotations of expressions in C (relative to M) is not merely some set T_C defined in terms of M, but is rather a set on which are defined boolean operations and a boolean relation. That is, T_C, *the type for* C (relative to M), is a boolean algebra. Such an approach is compatible with all model theoretic approaches, such as Montague Grammar, and is exemplified in *Logical Types for Natural Language* (KEENAN & FALTZ, 1978/81), henceforth LT.

The purpose of this paper is to present some of the advantages of formulating natural language semantics in this way, irrespective of what other model theoretic apparatus is used. Section 1 below presents some basic concepts of boolean algebra, and Sections 2 - 4 the advantages: 2: simplifying the ontology implicit in the model, and a suggestion for a new approach to intensional properties; 3: extending the class of expressions within a category which can be *directly* interpreted, and a consequent new approach to presupposition; and 4: enriching the class of categories which are treated in the logic, and the consequent possibility of stating universal constraints on the logical form of natural languages which are not apparent (though not necessarily unstatable) on non-boolean approaches.

Finally, this paper supports a further very general claim. First, I note without argument that compared to many commonly studied algebras such as groups, lattices, and rings, boolean algebras possess a particularly rich structure, sufficiently much that it is surprising that any category of natural language is semantically boolean. Second, this paper, and in much more detail, LT(78/81) show that very many categories of natural language are semantically boolean, so the boolean nature of natural language is not category specific. And this suggests, as BOOLE (1847) felt, that the boolean operations represent "laws of thought", properties of the way we understand the world.

1. BOOLEAN ALGEBRA

In general, an algebraic structure is a non-empty set, the *domain* of the algebra, on which are defined various operations (functions) and relations satisfying certain conditions (axioms). I shall first present a familiar example of a boolean algebra and then give the general definition.

Consider as the domain of a boolean algebra the power set of a non-empty set X, that is the set whose members are just the subsets of X. Denote this set P(X). It has at least two members, $\emptyset$ (the empty set) and X itself (taken to be non-empty). And for any $A \in P(X)$ we have that $\emptyset$ is a subset ($\subseteq$) of A and $A \subseteq X$. In this sense $\emptyset$ is the least or zero (0) element of the domain and X itself the greatest or unit (1) element. Further, for all A and B in P(X) we have that $A \cap B$ and $A \cup B$ are subsets of X and thus in P(X). So intersection and union are binary operations defined on P(X). And they have certain characteristic properties, e.g. they are *commutative*, $A \cap B = B \cap A$, and ditto for unions; and each *distributes* with respect to the other, e.g. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$, and ditto interchanging the intersection and union symbols. Moreover, $A \cap B$ is a *lower bound* for the set $\{A,B\}$ in that it is a subset of each member of $\{A,B\}$. In fact, it is the *greatest lower bound* in that for all $Y \in P(X)$, if Y is a subset of each of A and B then Y is also a subset of $A \cap B$. Similarly, $A \cup B$ is the least upper bound for $\{A,B\}$. (Each of A and B is a subset of $A \cup B$ and if that also holds for Y then $A \cup B$ is a subset of Y.) Note further that $A \subseteq B$ iff $A \cap B = A$. So, perhaps perversely, we could actually define the subset relation in this way.

Finally, we define on P(X) a one place operation called (absolute) complement and denoted ' as follows: For all $A \in P(X)$, A' is the set of those objects in X which are not in A. Clearly $A' \subseteq X$ so $A' \in P(X)$. And note the following obvious truths, for all $A \in P(X)$: $A \cap A' = \emptyset$, $A \cup A' = X$, $A \cap \emptyset = \emptyset$, and $A \cup X = X$.

More generally we define a boolean algebra $\mathcal{B}$ to be a (horrendous) 7-tuple consisting of a non-empty set B (the domain), two elements $0_\mathcal{B}$ and $1_\mathcal{B}$ of B called the zero and unit elements, two binary operations on B, $\wedge_\mathcal{B}$ and $\vee_\mathcal{B}$ called *meet* and *join*, respectively, one unary operation $'_\mathcal{B}$ called *complement*, and a binary relation $\leq_\mathcal{B}$ called *less than* or *equals*. $\mathcal{B}$ is required to satisfy the following axioms (omitting here and elsewhere the subscripts) for all x,y,z in B:

(a) Commutativity

$(x \wedge y) = (y \wedge x)$ and $(x \vee y) = (y \vee x)$.

(b) Distributivity

$(x \wedge (y \vee z)) = (x \wedge y) \vee (x \wedge z)$ and $(x \vee (y \wedge z)) = (x \vee y) \wedge (x \vee z)$.

(c) Laws of Complements

$(x \wedge x') = 0$ and $(x \vee x') = 1$.

(d) Laws of Zero and Unit

$(x \wedge 0) = 0$ and $(x \vee 1) = 1$.

(e) $0 \neq 1$.

(f) $x \leq y$ iff $(x \wedge y) = x$.

This axiomatization (adapted from MENDELSON, 1970) is quite redundant. (f) is just a definition of $\leq$ and either $\wedge$ or $\vee$ could be defined in terms of the other plus complement since the De Morgan laws (e.g. $(x \wedge y)' = (x' \vee y'))$ follow from the axioms. It also follows that $\leq$ behaves like the subset relation, e.g. $x \leq x$, if $x \leq y$ and $y \leq z$ then $x \leq z$, and if $x \leq y$ and $y \leq x$ then $x = y$. And it follows that $x \wedge y$ is the greatest lower bound for $\{x,y\}$ and $x \vee y$ is its least upper bound.

Given the definition we could now prove that for any non-empty set X, $\langle P(X), \cap, \cup, ', \emptyset, X, \subseteq \rangle$ is a boolean algebra (called a power set algebra). Another useful boolean algebra is $\langle 2, \wedge, \vee, ', f, t, \leq \rangle$ where 2 is the set $\{t,f\}$ of truth values and $\wedge$, $\vee$, and ' are defined by the standard truth tables for conjunction, disjunction, and negation, respectively. It follows that $t \leq t$, $f \leq t$, $f \leq f$, and $t \nleq f$. A further example, useful in intensional logic, is the entire set of functions from a non-empty set J (say a set of possible worlds) into B, the domain of some boolean algebra. Denote this set $F_{B/J}$. The boolean operations are *defined pointwise* on J, by which is meant that for f and g any functions in the set, $(f \wedge g)$ is that function which maps each j in J onto $f(j) \wedge g(j)$, where the meet sign on the right refers to meets in B, since f(j) and g(j) are elements of B.

Similarly $(f')(j) =_{df} (f(j))'$. The zero function maps each j onto the 0 element of B (we write, omitting subscripts, $0(j) = 0$) and similarly $1(j) = 1$. It follows that $f \leq g$ iff for all j in J, $f(j) \leq g(j)$. Note that J can be any set, in particular a set of n-tuples of places, times, etc.

For purposes of Section 2 (which is self contained and may be omitted on first reading as it is somewhat more algebraic than the other sections) two further concepts of boolean algebra will be needed. First, an algebra is said to be *complete* if for every subset K of its domain B there is an element x in B which is the greatest lower bound for K (i.e. $x \leq$ everything in K, and if $y \in B$ meets this condition then $y \leq x$). For a complete algebra the element x referred to above is denoted $\bigwedge K$. Analogously for $\bigvee K$. Note that power set algebras are complete. For $K \subseteq P(X)$, $\bigwedge K$ is just $\bigcap K$, the set whose members are just those elements in all of the k in K.

Second, an *atom* of an algebra is an element of the domain which is not the 0 element but which no other element is strictly less than. That is, b is an *atom* in B iff for all x in B, if $x \leq b$ then $x = 0$ or $x = b$. And an algebra is said to be *atomic* if for all non-zero elements y in the domain there is at least one atom b such that $b \leq y$. Any power set algebra is atomic, the unit sets being the atoms. (Similarly the algebra 2 is complete and atomic, and the pointwise algebra $F_{B/J}$ is complete and atomic whenever B is the domain of a complete and atomic algebra.)

Finally, I note from standard boolean algebra that any complete and atomic algebra is isomorphic to a power set algebra (the function mapping an element onto the set of atoms $\leq$ to it is an isomorphism of the algebra onto the power set of its atoms). So up to isomorphism the complete and atomic algebras are the power set algebras.

2. SIMPLIFYING THE ONTOLOGY

2.1. One of MONTAGUE's (e.g. 1972) linguistically most useful insights was his definition of the types (relative to a model) which enabled us to treat singular terms - *John, the king of France*, etc. - as taking their denotations in the same type as quantified NPs like *every man*. Thus the logical forms for *John walks* and *every man walks* are identical up to the difference in internal structure of the logical forms for *John* and *every man*. Thus we have a better explanation than in standard logic (SL) for how we interpret English sentences as a function of their form; in SL *every man* is not even assigned a logical form at all.

Let us review briefly the essence of Montague's innovation here. First, in SL a model (the ontological primitives) is a pair <2,U>, where 2 is the set of truth values and U is the set of "things that exist". In both cases we have a reasonable pretheoretical idea what the elements of these sets are intended to be. In particular (ignoring 2 as it is "standard") U is the type for the individual constants (≈ proper nouns) and the range of the individual variables, so it may contain things like you, and me, and John, etc. While we might debate whether U and 2 should be taken as *primitives* it is clear that these sets do comprise parts of our ontology and it is not surprising that other categories should have their types given as a function of these. For example, the type for the 1-place predicates in SL will just be the subsets of U, i.e. P(U), or equivalently the set of functions from U into 2. Let us call these sets (or functions) *extensional properties*.

Montague's innovation here was to treat the denotations of proper nouns (PNs) not as elements of U, but rather as sets of extensional properties, called for the nonce *individuals*. More formally, for all $x \in U$ we define the *individual determined by* x to be $\{K \subseteq U: x \in K\}$. So an individual is defined in terms of an element of U. And being a collection of sets of properties (I drop "extensional"), an individual is a subset of P(U), that is a member of P(P(U)). So the type for full NPs like *John* and *every man* is P(P(U)) and that for common noun phrases (CNPs) is P(U). And *every man* will be interpreted as the intersection of the individuals which have the *man* property; *a man* as their union, etc. and *John, every man, a man*, etc. may be treated as expressions in the same gross category. (I say 'gross' because PNs will not be interpreted as arbitrary members of T_{NP} but may only have individuals as their possible denotations, so they are in effect a distinguished subcategory of NP. Note that an intersection (union) of distinct individuals is never an individual.)

But the nice ontological character of SL has been lost, for a model of this system is still a pair <2,U> and while 2 is still the truth values, the elements of U are now not possible denotations for any expressions in English. Hence we have no pretheoretical idea what the elements of U are, and it is mysterious why other expressions, e.g. *John, every man*, and indeed most other expressions, should have their denotations given as a function of elements of U. Notice that the "things that exist", e.g. denotations of PNs, are the individuals, not elements of U. So U seems to be a kind of noumenal world underlying the phenomenological world of individuals.

And our ontological qualms are not assuaged in the slightest by noting that U and the set of individuals are in a natural one-to-one correspondence (so are the even numbers and the odd numbers, but they have very different properties). Such a correspondence (onto) just says the two sets have the same size. But they crucially fail to have the same structure. For individuals, being sets, are the kinds of things we can take boolean combinations of, e.g. intersections, unions, etc. And this we must do (regardless of how the statement is actually formulated) in order to get denotations for *every man*, *a man*, etc. Moreover, there is no way to assign a boolean structure to an abitrarily chosen set U. For example, all finite boolean algebras have 2^n elements for some finite n. So there are no boolean algebras with 3, or 6, or 7 elements. In fact, closing the individuals under arbitrary intersections and unions gives us P(P(U)), the set of all the sets of properties, much larger than U.

From this point of view part of Montague's innovation lay in trading in the elements of U for things which we can treat in a boolean way. And once this is recognized there is a very easy way to extend his insight so as to eliminate the ontological qualms above. This extension not only yields a new ontology, but it generalizes in ways that permit a new and potentially more adequate approach to the treatment of "hyper-intensional" CNPs, e.g. *imaginary horse, book that John intended to write but never wrote*, etc.

As a first step in the extension notice that if we take T_{CNP} as P(U) we may automatically regard it as a complete and atomic (ca) algebra its atoms being the unit sets of elements of U. And the individual determined by x, namely $\{K \subseteq U: x \in K\}$ is $\{K \in P(U): \{x\} \subseteq K\}$, as trivially $\{x\} \subseteq K$ iff $x \in K$. But $\{x\}$ is an atom of P(U) and $\subseteq$ is just the boolean relation on P(U), so this last set basically defines an individual in terms of the boolean structure of $P(U) = T_{CNP}$. So our first step is the following preliminary definitions:

<u>PRELIMINARY DEFINITION 1</u>. A *model for* L is a pair <2,P>, where 2 is as before and P is any complete and atomic boolean algebra.

<u>PRELIMINARY DEFINITION 2</u>. For each atom b in P, I_b, *the individual determined by* b, $=_{df} \{p \in P: b \leq p\}$.

<u>REMARKS</u> on the preliminary definitions:

1) They do appear to constitute a *new* ontology, for now (extensional) properties and truth values are the ontological primitives, not entities or individuals and truth values.
2) There appear to be no mysteries in the ontology since each primitive is the type for some category of expression. In particular, the elements of P are the kinds of things that expressions like *man, tall man, man who Mary loves*, etc. can denote. And while we shall want to query further what their exact nature is, we will know how to reason about them since we know what ordinary expressions they are the intended interpretations of. And
3) up to isomorphism, the class of possible types for CNP and hence of individuals is the same as on Montague's earlier view. For obviously if T_{CNP} is P(U) for some U then it is a ca-algebra and the above definition picks out the same sets as individuals as the earlier definition. And if P is not specifically a power set algebra it is, by the remark at the end of Section 1 isomorphic to the power set of its atoms, so taking the set of atoms as U we have a type for CNP in the old sense, one that is isomorphic to the given one. Thus any arguments which would be shown valid on the new approach are valid on the old and vice versa. So the two approaches are descriptively adequate to the same extent.

Let us consider some objections to this approach, ones that will revise somewhat our preliminary definitions. First it has been objected that this approach is just a "mathematical trick". But that is silly. It is neither more nor less mathematical or tricky than Montague's observation that the elements of U are in a one-to-one correspondence to the individuals as defined on that approach.

More seriously however one can query whether this approach really constitutes a *new* ontology or whether it just gives us the same one in different mathematical garb. To be more precise: What motivation do we have for taking T_{CNP} (= P) as a ca-algebra other than our desire to treat it as we always did, namely as a power set algebra? And second, while P itself may not generally be mysterious, individuals are defined above in terms of atoms, and are not these properties every bit as mysterious as the elements of U on the old approach?

I will answer these objections as follows. First I will show that we have independent motivation, in terms of correctly representing our judgments of logical truth and entailment on English, for taking P as a complete boolean algebra. Similarly I will show that, taking NP denotations as subsets of P, we have direct motivation for requiring that PN denotations be subsets of P which meet certain conditions, and that when we define individuals as the subsets of P which meet those conditions we get the individuals in the old sense. Neither the judgments of validity and entailment nor the formal conditions mention or in any way presuppose the notion of an atom or that P is atomic. Thus the notion of individual is conceptually and formally independent of that of an atom. And third I will show that there is independent motivation for requiring that P have atoms; specifically that to correctly represent the valid arguments on English there are property denoting expressions which should be and are intended to be interpreted as atoms. So atoms are not particularly mysterious.

But this is as far as the independent motivation for the boolean nature of P will go. And if we merely require that P be complete and have atoms but not be atomic (which would require in addition that for every non-zero q in P there is an atom b ≤ q) we obtain a properly larger class of T_{CNP}'s than in the extensional systems of Montague or LT, and this larger class is rich enough to provide denotations for the hyperintensional CNPs mentioned above. So in fact what appears to be the descriptively most adequate approach here does not exactly reconstruct the systems of PTQ or LT. If of course we impose the additional requirement that P be atomic we do obtain the earlier systems.

2.2. T_{CNP} should be a boolean algebra

Our intent is that P (= T_{CNP}) provide denotations for expressions like *man*, *socialist*, *vegetarian*, etc. among others. We assume that full NPs like *John* and *every man* will be interpreted as subsets of P. So we want our semantics to guarantee that sentences like *John is a socialist* are true in A = <2,P> iff the subset of P which interprets *John* has the property which interprets *socialist* as a member. Now consider sentences like (1):

(1) John is both a socialist and a vegetarian.

We want (1) to be true in <2,P> iff the John set of properties has the property of "being both a socialist and a vegetarian". But which property is that? Clearly it is not arbitrary relative to which elements of P interpret *socialist* and *vegetarian* (call them p and q, respectively). Arguably the property should be (p ∧ q); What is the argument? Well, one argument is that since meet is commutative and thus (p ∧ q) is the same element of P as (q ∧ p), this analysis predicts that (1) should be logically equivalent to *John is both a vegetarian and a socialist* since our semantics will say in each case that the same property is in the John set. And this is pretheoretically judged correct. A similar argument shows that (p ∨ q) should be the property of "being either a socialist or a vegetarian".

These two claims jointly make more (correct) predictions. Since e.g. meets distribute over joins we have that (p ∧ (q ∨ r)) is the same element of P as (p ∧ q) ∨ (p ∧ r). Thus (2a) and (2b) should be judged logically equivalent, and they are.

(2) a. John is both a socialist and either a vegetarian or a cannibal
b. John is either a socialist and a vegetarian or a socialist and a cannibal.

Further, judgments on similar sentences force constraints on what sets of properties are admissible interpretations for PNs like *John*. Thus (3a) and (3b) are judged logically equivalent.

(3) a. John is both a socialist and a vegetarian
b. John is a socialist and John is a vegetarian.

Thus John must be a set of properties such that for all p,q in P, both p and q are in John iff (p ∧ q) is. Note that many reasonable sets fail this condition (which we call *strongly closed under meets*). E.g. replacing *John* in (3) everywhere by *no student*, it is obvious that (3a) does not entail (3b). So (p ∧ q) can be in no student without it necessarily being so that both p is in it and q is in it. So John cannot be interpreted by the kinds of property sets which interpret no student.

Similarly replacing *and* in (3) everywhere by *or* we can infer that whenever (p ∨ q) is in John then either p is or q is, and conversely, since the a- and b-sentences are again judged logically equivalent. And again many reasonable sets of properties, such as those denoted by *every student*, need not meet these conditions, since logical equivalence fails in (3) if *John* is everywhere replaced by *every student* (and *and* by *or*).

Now consider the trickier case of complements. Our pretheoretical judgments (accepting the two valued nature of the system, something which is easily modified on a boolean approach but not something I am modifying here) tell us that in any state of affairs exactly one of (4a) and (4b) below are true:

(4) a. John is a man

b. John is not a man.

So the conjunction of these two sentences must be logically false and their disjunction logically true. All these judgments are correctly predicted if the property of "being not a man" is taken as the complement of the property "being a man" and we require of any possible PN denotation I that for any property p, I contains exactly one of {p,p'}. I shall then impose these conditions in order to correctly represent these judgments.

There is also direct motivation of a different sort for taking T_{CNP} as a boolean algebra. Consider the logical properties of extensional adjectives (APs) like *female* and *tall*. Such APs will be interpreted by functions from P into P, and if *female* is such a function f and *socialist* is a property p, then *female socialist* will be interpreted by f(p), the value of f at p. But the property f(p) is not arbitrarily related to p. E.g. *Mary is a female socialist* entails *Mary is a socialist*. So we will want to require that the functions f which can interpret extensional APs meet the condition that for all p in P, $f(p) \leq p$. That is, $f(p) \wedge p = f(p)$ so "being both a female socialist and a socialist" is not different from "being a female socialist". And the requirement that PN denotations be strongly closed under meets guarantees that whenever Mary has the property f(p) then she has the property p. For f(p) is $f(p) \wedge p$ and by strong closure we infer that Mary has p.

So another reason for taking T_{CNP} as boolean is that we want to use the boolean $\leq$ relation on T_{CNP} to correctly characterize certain valid arguments involving extensional APs.

INTERIM CONCLUSION: If we take T_{CNP} as a boolean algebra and constrain the interpretations of PNs in the ways indicated we correctly represent many valid arguments and logical truths (assuming the basic two valued nature of the system). So we have independent motivation for taking T_{CNP} as boolean; and we have not covertly relied on any notion of an atom nor have we in any way assumed that T_{CNP} will be isomorphic to a power set algebra.

2.3. T_{CNP} (=P) is a complete algebra

Consider the following valid argument: *Mary is taller than every man, John is a man*; therefore *Mary is taller than John*. Assuming the analysis so far, what property must Mary have above for the argument to be valid? Well, letting M be the set of PN denotations with the *man* property, and for each m_i in M letting tm_i be the property of "being taller than m_i", we want Mary's property to be: $tm_1 \wedge tm_2 \wedge \ldots$ for each m_i in M. But this is just what is meant by $\wedge\{tm_i: m_i \in M\}$. So if we take P as complete we will have denotations for property denoting expressions like *taller than every man*. And if we require of PN denotations that they be strongly closed under arbitrary meets, not just the binary ones mentioned earlier, then the above argument is shown valid. So I shall take P as complete, and define individuals (PN denotations) by:

DEFINITION 1. For P any complete boolean algebra, I is an *individual*[1] on P iff I is a subset of P satisfying (i) - (iii) below:

(i) *Completeness*: for all p in P, either $p \in I$ or $p' \in I$.

(ii) *Consistency* : for all $p \in P$, not both $p \in I$ and $p' \in I$.

(iii) *Meets* : for all $K \subseteq P$, $K \subseteq I$ iff $\wedge K \in I$.

As Definition 1 does not mention the notion of an atom, or even require that P have atoms, it is clear that individuals are conceptually and formally independent of that of atoms. Theorem 1 below may then seem surprising:

THEOREM 1. I is an individual on P iff for some atom $b \in P$, $I = \{q \in P: b \leq q\}$.

PROOF. Suppose that I is an individual on P. We show that $\wedge I$ is an atom and that $I = \{q: \wedge I \leq q\}$, thus proving the first half of the theorem.

(a) 0 (the zero element of P) is not in I. Otherwise, since $0 = 0 \wedge 1$ we have that $0 \wedge 1 \in I$, so from (iii) $1 \in I$. But $1 = 0'$, so both 0 and 0' are in I, contradicting (ii). So $0 \notin I$ (and by (i), 1 is in I, so I is not empty).

(b) If $p \in I$ and $p \leq q$ then $q \in I$. By assumption $p = (p \wedge q)$, so $(p \wedge q) \in I$, so by (iii) $q \in I$. (More exactly: $\{p,q\} \subseteq I$, whence $q \in I$.)

(c) $\{q: \wedge I \leq q\} \subseteq I$. By (iii) $\wedge I \in I$ (since $I \subseteq I$), so from (b) if $\wedge I \leq q$ then $q \in I$.

(d) $I \subseteq \{q: \wedge I \leq q\}$. Let $p \in I$. By definition of $\wedge$, $\wedge I \leq p$.

(e) $I = \{q: \wedge I \in q\}$. Immediate from (c) and (d).

(f) $\wedge I$ is an atom. Suppose otherwise. Then from definition of atom, either $\wedge I$ is 0 or there is a non-zero $p < \wedge I$. $\wedge I$ is not 0 since $0 \notin I$ and from (iii) $\wedge I \in I$. So let p such that $0 < p < \wedge I$. Then $\wedge I \nleq p$ so from (d) $p \notin I$. But also p' is not in I. For otherwise from (d) $\wedge I \leq p'$, whence by transitivity of $\leq$, $p \leq p'$, thus $p \wedge p' = p$. But $p \wedge p' = 0$, contradicting that $p \neq 0$. So p' is not in I. So neither p nor p' are in I, contradicting the assumption that I is an individual. Thus $\wedge I$ is an atom.[3]

The other half of the theorem is straightforward. Thus, assume that b is an atom of P and let $M = \{q: b \leq q\}$. We show M is an individual:

(a) *Meets*: first, let $K \subseteq M$. So each k in K is in M, so $b \leq$ each such K. So b is a lower bound for K. But $\wedge K$ is the greatest lower bound, so $b \leq \wedge K$, thus $\wedge K$ is in M.
Second, suppose $\wedge K \in M$. So $b \leq \wedge K$, and since $\wedge K \leq k$, all k in K, we have by transitivity of $\leq$ that $b \leq k$ all k in K, so all k in K are in M, so $K \subseteq M$.
Thus $K \subseteq M$ iff $\wedge K \in M$.

(b) *Consistency*: suppose both p and p' in M. Then $b \leq p$ and $b \leq p'$, so $b \leq (p \wedge p') = 0$, contradicting that b is an atom.

(c) *Completeness*: suppose $b \nleq p$. Then $(b \wedge p') \neq 0$. Since b is an atom, then $(b \wedge p') = b$, so $b \leq p'$. So for any p, $b \leq p$ or $b \leq p'$, so either p is in M or p' is. □

Theorem 1 together with Definition 1 tell us that *if* we take P as atomic (and complete of course) then the individuals will be just the sets they were on the earlier definition. So if P = P(U) for some U, the individuals are just the subsets K of U which contain a fixed element of U. However, neither Definition 1 nor Theorem 1 presuppose the existence of atoms, much less that P is atomic and thus (isomorphic to) a power set algebra. If P in Theorem 1 were selected as complete and atomless (there are such algebras) then there would be no individuals on P. Moreover, if P were selected to have atoms but still not be atomic then the individuals on P would still be just the subsets of P which dominate a fixed atom, and thus still have all the properties guaranteed by the definition of individual. Note further (see KEENAN (to appear b) for a proof) that from standard boolean algebra we have that for any cardinal n there are non-atomic complete algebras with exactly n atoms. So we may have as many ordinary individuals as we like without requiring that P be atomic. So the atomicity of P, which is forced in e.g. PTQ and LT, remains an open question.

We do however want to require that P have at least some atoms. There are at least two reasons for this. First, if P has no atoms then by Theorem 1 it has no subsets which meet the condition for being an individual. But we want such subsets in order to provide interpretations for PNs like *John* so that the logical truths and valid arguments mentioned earlier can be shown to be valid.

And second, it follows from Definition 1 and Theorem 1 that there is a one-to-one correspondence between the atoms of P and the individuals on P. (No individual can contain two different atoms, for then it contains their meet, which is 0, and no individual contains 0 by (a) above). So an atom is an extensional property[2] which exactly one individual has. And there are many property denoting expressions in English intended to be interpreted by such properties: *tallest student, first* (*third*, etc.) *man to set foot on the moon, student who stood exactly here at exactly noon yesterday, man who is the only man that Mary loves,* and even *doctor who is John,* etc. Of course, such expressions might fail to denote atoms (if e.g. the two tallest students had the same height then none would have the tallest student property). But clearly such expressions cannot denote properties that more than one individual has. So if P had no atoms these expressions would *have to* denote properties which no individual has, and that is clearly wrong.

CONCLUSION. We have not taken P as complete and atomic in order to, in effect, treat it as the power set of some set (the universe of discourse). In fact, we have not taken P as atomic, but only required that it have some atoms. Moreover, the notion of an individual is conceptually and formally independent from that of an atom, and atoms are not mysterious. They are the intended denotations for many common expressions. We refer the reader to KEENAN (to appear a) for a more detailed discussion of this argument.

2.4. A new approach to hyperintensional CNPs

What are the principle differences between atomic and non-atomic P's (assumed complete without further statement) and what then is the evidence for or against taking T_{CNP} as atomic? Here we note just one such property and refer the reader to KEENAN (to appear b) for proofs of the claims made below and a much more thorough discussion. The principle diffference is this: If P is atomic (and complete), then p and q are the same elements of P iff they are members of exactly the same individuals. So if p is different from q then there is an individual which contains one but not the other. This condition fails however for non-atomic algebras. We may have distinct properties in exactly the same individuals. Query: Do we want this?

The prima facie case is overwhelmingly yes. Intuitively we do not want to say that *doctor* and *lawyer* are the same property even if the individuals with one are just those with the other. But of course "standard" intensional logic (SIL) has addressed this problem as follows. Let us think of T_{CNP} (in an intensional logic) as the set of functions from possible worlds J into extensional properties, i.e. as $F_{P/J}$. And if we interpret *doctor* and *lawyer* by functions in this set then obviously they may have the same values (extensions) at some of the j in J but still be different functions as long as they do not have the same value at all j in J. And this answers the prima facie problem, though it does seem funny that in some models, now taken as triples <2,P,J>, *doctor* and *lawyer* are interpreted as the same elements of $F_{P/J}$. That is, in some models they have the same intension and in others they do not. We rather think of the intension of a CNP as constant, not varying with how the world is.

But the standard approach, as has been recognized, is not sufficiently general. Thus if P is complete and atomic, any two CNPs which of necessity have the same extension (same value at j) in every possible world must have the same intension, that is be the same function from J into P, regardless of what J and P are chosen. But there are many examples of such CNPs which still nonetheless should be interpreted as different properties, e.g. *imaginary horse, imaginary lion, book that John intended to write but never wrote and never will write*, etc. And clearly no individual, such as me or you or my horse, can have the property expressed by *imaginary horse*. And since, in a complete and atomic P, there is only one element that no individual has, namely the 0 property, the extension of *imaginary horse* in each j in J must be the 0 property. Ditto for *imaginary lion*, etc. Hence on the standard view the hyperintensional CNPs must always be interpreted by the same element of $F_{P/J}$, that is have the same intension. So sentences like *an imaginary horse is an imaginary lion*, etc. will be valid, which is obviously wrong.

But if we do not require that P be atomic we may correctly represent different hyperintensional CNPs by different properties with the same extension (in particular the 0 extension) without recourse to a possible world semantics at all. Thus not requiring P to be atomic gives us the potential for correctly representing valid arguments which are incorrectly represented on standard approaches. Notice of course that taking P as complete, with atoms, but non-atomic is a move completely independent of possible worlds representations for CNPs. As that approach does seem to correctly represent at least certain logical notions of necessity and possibility we could on the suggested approach still take T_{CNP} in the intensional logic as $F_{P/J}$, we are merely requiring that P be non-atomic. And we do not need to use the j's to distinguish *imaginary lion* from *imaginary horse* but we can still use them to distinguish say a *possibly Albanian diplomat* from a *necessarily Albanian diplomat*.

If this approach to such irrealis APs as *imaginary, unreal, pretend, make-believe* and perhaps *mythological* and *fictional* proves viable it will constitute strong motivation for a boolean approach to semantics, as it would have been inconceivable had we not been taking T_{CNP} as a boolean algebra in the first place.

3. EXTENDING THE CLASS OF DIRECTLY INTERPRETABLE EXPRESSIONS

3.1. The most obvious advantage of our boolean approach is that we have a general - but not infallible! - way to interpret conjunctions, disjunctions, and negations of expressions in a given category; namely as the meets, joins, and complements respectively of the interpretations of the conjuncts, disjuncts, and "negatees". Thus we need not pretend that the boolean connectives (*and, or, not*) "really" only apply to sentences and "translate" sentences containing boolean combinations of non-sentences into ones where all boolean combinations are sentences. Notice that this is the same type (though in a sense lesser in magnitude) of advantage as Montague's original proposal. There is no particular difficulty in translating *every man walks* into $(\forall x)(\text{man}(x) \rightarrow \text{walk}(x))$ but if we do we are saying that the obvious syntactic structure of the English sentence is not the one we use to assign it a meaning, and we are left with the problem of explaining how a "right" logical form is learned given all the possible ones which differ from the "right" one only by logical operators.

Similarly on the boolean approach taken in LT, sentences like *John read Ulysses*, *every student read a book*, and *every teacher both read and criticized some book* have identical logical forms up to the difference in internal structure of the NPs and the TVP. So we have a better account on a boolean approach of how sentences are assigned a meaning as a function of their form.

Perhaps more important, we have a better account, at least an account, for why *and*, *or*, and *not* should be usable so freely in forming complex expressions in most categories (*sing and dance*, *some but not all*, *dishonest or careless*, *can and should*, *both in and behind*, etc. Namely *and*, *or*, and *not* are always interpreted as *meets*, *joins*, and *complements*. That they are meets, etc. of course depends on what their arguments are. So one can imagine that on the basis of a few simple examples, *sing and dance*, *John and Mary*, etc. one learns the basic boolean properties of the boolean operators, and then extends them naturally to other categories, even in fact to collocations which are not natural categories, as in *every diligent but not necessarily every intelligent student will pass*.

Note of course that interpreting the boolean connectives as the appropriate boolean functions makes very strong predictions concerning the logical behaviour of the expressions in question, ones that are often but not always borne out. For example, not all uses of *and* in English appear to be commutative (cf. *Mary got pregnant and (then) married* vs. *Mary got married and (then) pregnant*). (For more interesting cases see Section 4.)

A second advantage here is that we can directly interpret negation in all categories (*is bald/isn't bald*, *a solid but not very pretty house*, *near but not on the table*, *some but not all*, etc.). So in particular we have a distinction between VP negation and sentence negation, and can thus handle the basic cases of presupposition without recourse to multivalued logics, supervaluations, etc.

3.2. A new approach to presupposition

Using the boolean representations of LT, all the sentences below except (5d) entail (5e).

(5)
- a. (The king of France)(be bald)
- b. (The king of France) [not (be bald)]
- c. (The king of France) [be (not bald)]
- d. (not [(the king of France)(be bald)])
- e. (The king of France)(exist).

To see that the relevant entailments hold, consider first that extensional (transparent) VPs like *be bald* are booleanly speaking structure preserving functions, that is, homomorphisms. More explicitly, we say that a function f from a (boolean) algebra B into an algebra D *preserves meets* iff for all x,y in B, $f(x \wedge y) = f(x) \wedge f(y)$, where the meet on the right of course refers to meets in D since f(x) and f(y) are elements of D. And to see that VPs like *be bald*, constructed as function from T_{NP} (= $P(T_{CNP})$ and thus a power set algebra) into T_S (= 2, the algebra referred to in Section 1) should be constrained to preserve meets, note e.g. that *John and some teacher are bald* must have the same truth value as *John is bald and some teacher is bald*. Similarly we say for f as above that f *preserves complements* iff for all x in B, $f(x') = (f(x))'$. And to see that semantically *be bald* preserves complements notice that *(not(every student)) is bald* must have the same truth value as *it is not the case that every student is bald*. We now define:

<u>DEFINITION 2</u>. h from B into D is a *homomorphism* iff h preserves meets and h preserves complements.

It follows from the (standard) definition that h preserves joins, since $(x \vee y) = (x' \wedge y')'$. Similarly, if $x \leq y$ then $h(x) \leq h(y)$. Further h maps the unit in B onto the unit in D, and ditto for the zero in B (onto the zero in D). To see the last point note that $h(0_B) = h(0_B \wedge 0'_B B) =$ $= h(0_B) \wedge h(0'_{BB})$, since h preserves meets, $= h(0_B) \wedge h(0_B)'_D$, since h preserves complements, $= 0_D$, since the meet of any element, even $h(0_B)$, with its complement is the zero element of the algebra.

Second, consider the natural semantics for *the*. It maps properties onto sets of properties as follows: *the*(p) is the unique individual which has p if there is one, and otherwise it is the zero element of T_{NP}, that is the empty set. More exactly, *the*(p) is the individual determined by p if p is an atom, and the empty set otherwise. (So we have another motivation for wanting P to have atoms, if it did not *the*(p) would always be $\emptyset$.)

Third, the (transparent) homomorphisms, e.g. *be bald*, etc. themselves form a natural algebra (we want to interpret expressions like *be bald and not be old*, etc. as the relevant meets and complements of VP denotation. And it turns out that a VP homomorphism of this sort is defined by stating its values on the individuals! See LT for a proof. That is, for any function from the individuals into 2 there is a unique (complete) homomorphism from $T_{NP} = P(T_{CNP})$ into 2 having just those values on the individuals. And in particular if h is a VP homomorphism then (h') is that VP homomorphism which assigns to each individual I the opposite value from what h assigns it. Thus *(not(be bald))* will be true of John iff *(be bald)* is false of John, which is intuitively correct.

Now consider the entailments mentioned above. If either *be bald* or *not(be bald)* hold of the denotation of *the king of France* then that denotation is not the zero element since *be bald* etc. are homomorphisms and map zero elements onto the zero element (f) in 2. And if *the king of France* denotation is not the zero element it is an individual and thus has the existence property. So both (5a) and (5b) entail (5e). Notice that (5d), sentence negation, will not entail (5e) since (5d) will be true just in case (5a) is false, and if France has no king then (5a) is false. So sentence and VP negation are non-trivially different at this point.

And this suggests the following definition of presupposition, using $H_{D/B}$ to denote the set of homomorphisms from B into D:

<u>DEFINITION 3</u>. For all b in B, d in D, h in $H_{D/B}$, if $H_{D/B}$ is a boolean algebra then the pair <h,b> logically *presupposes* d iff $h(b) \leq d$ and $(h')(b) \leq d$.

And by extension we may define:

<u>DEFINITION 4</u>. For S and T sentences of L, S *presupposes* T iff S is of the form (np,vp) and for all interpretations m of L, <m(vp),m(np)> logically presupposes m(T).

Definition 4 may seem insufficiently general in that it only applies to S's of the subject-predicate form, and not say to ones that are conjunctions of other S's, etc. But the definition does appear to capture the clearest case of presupposition in the literature and it nowise precludes any extensions to larger classes of S's.

On the other hand, one may doubt whether Definition 4 will apply to the other clearest case in the literature, namely that factive sentential predicates presuppose their sentential subjects. It would *appear* that such predicates are not homomorphisms as assumed in Definition 4. E.g. (6a) does not even entail (6b), much less preserve meets.

(6) a. That John passed and Mary failed is strange
b. That John passed is strange and that Mary failed is strange.

In fact, however, there is an independently motivated analysis of factive predicates which does treat them as homomorphisms and which accounts for an ambiguity and some entailments not, to my knowledge, previously noticed. I sketch the analysis here and refer the reader to LT (to appear) for details.[1]

Take (standardly) the intensional type for S to be the set of functions from J, the set of possible worlds, into 2. By pointwise definition on J it is a boolean algebra, isomorphic to the power set of J. Denote this set Pr (for proposition). A *property of a proposition* will be a function assigning each proposition, i.e. each element of Pr, a truth value. And for each p in Pr, define I_p to be the set of properties assigning p value t. Interpret *that* (complementizer) in English as a function mapping each p in Pr onto I_p, and note that *that* is one-to-one from Pr onto the set of I_p's. And take the type for sentence complements in general to be all the sets obtained by taking arbitrary intersections, unions, and complements of the I_p's. That set is provably the set of all sets of properties of propositions, i.e. P(P(Pr)), a power set boolean algebra. Of course conjunctions, disjunctions, and negations of *that*+S's are just the meets, joins, and complements of the conjuncts, etc. as usual. So for example, in (6a) above *be strange* is predicated of $I_{(p \wedge q)}$ the set of functions which assign $(p \wedge q)$ value true, where p is the proposition which interprets *John passed* and q the one which interprets *Mary failed*.

In (6b) on the other hand, *be strange* is predicated once of I_p and once of I_q. Now given that neither p nor q are $\leq$ to the other, so p,q, and $(p \wedge q)$ are all distinct propositions, it follows that I_p, I_q, and $I_{(p \wedge q)}$ are different sets of properties. In particular $I_{(p \wedge q)}$ will contain that function which maps $(p \wedge q)$ onto t and everything else, e.g. both p and q, onto f. Hence, correctly, (6a) on this account will not entail (6b).

Notice now that all sentence complement taking predicates are homomorphisms! Thus (7a) and (7b) will be logically equivalent, and this judgment is correct:

(7) a. Both that John passed and that Mary failed are strange
b. That John passed is strange and that Mary failed is strange.

Similarly, this analysis predicts the logical equivalence of (8a) and (8b).

(8) a. That John passed and not that Mary failed is strange
b. That John passed is strange and it is not the case that it is strange that Mary failed.

Notice also that elements in the type for sentence complements ($\bar{S}$) will be denoted by expressions other than *that*+S's and their boolean combination. At a first guess for example *everything* in *John believes everything* should be the intersection of the I_p taken over all p in Pr. And *something that Harry said* might be represented as the union of the I_p which are such that Harry said I_p is true, etc. And the $\bar{S}$ taking predicates also behave homomorphically on such elements of $T_{\bar{S}}$, e.g. (9a) and (9b) below are judged logically equivalent:

(9) a. Something but not everything is strange
b. Something is strange but it it not the case that everything is strange.

(Many intriguing questions arise here concerning the binding of possible world indices, the validity of sentences like *John believes everything that he believes,* the entailment between *everything which is so is strange* therefore *the fact that everything which is so is strange is itself strange.*)

Consider now the specifically factive character of *be strange, ironic, surprising, pleasing,* etc. Clearly *that John passed is strange* entails *John passed*. So we want to restrict the possible interpretations of $\bar{S}$ taking VPs like *be strange* etc. to those functions from P(P(Pr)) into Pr which are both homomorphisms and *factive*, as defined in:

<u>DEFINITION 5</u>. A function f from P(P(Pr)) into Pr is *factive* iff for all p in Pr, $f(I_p) \leq p$.

Chosing *be strange* from the factive functions then we are guaranteed that the truth value of *that John passed is strange* in $j \leq$ the truth value of *John passed* in j, and thus the former sentence entails the latter.

Now consider boolean combinations of factive predicates. Clearly the a- and b-sentences below are judged logically equivalent:

(10) a. That John left early is both strange and unpleasant
b. That John left early is strange and that John left early is unpleasant.

(11) a. That John left early is not strange
b. John left early and it is not the case that it is strange that he did.

(10a) says that the value of a conjunction of factive predicates on a propositional individual is defined pointwise on the individuals (functions on the individuals extending uniquely to complete homomorphisms). Thus for f and g factive functions $(f \wedge g)(I_p) = f(I_p) \wedge g(I_p)$, meets on the right being taken in the algebra Pr of propositions.

More interestingly, (11) says that the complement of a factive predicate is still factive. That is, $(f')(I_p) = p \wedge (f(I_p))'$, (meets and complements on the right taken in Pr of course). It is straightforward to show that meets and complements of factive functions are factive, and that meet and complement as defined satisfy the axioms of boolean algebra. (Joins are defined pointwise on the I_p's like meets.)

Thus (treating $\bar{S}$'s as NP's) our definition of presupposition applies to factive predicates, since they are homomorphisms, and, moreover, yields the correct results: Both *that* P *is strange* and *that* P *(not is strange)* entail P. And *It is not the case that* [*that* P *is strange*] does not, as it only says that *that* P *is strange* is false, which it will be if P is false.

In summary, being able to take complements in the types for essentially all categories has enabled us to define a presupposition relation which captures the clear cases, leaves extensions to less clear ones open, and depends crucially on sentential and ordinary VPs being boolean categories with the consequent distinction between VP and S negation. Moreover, our judgments of presupposition here are accounted for by the independently motivated assignment of boolean structure to the categories. No additional apparatus (3 truth values, gaps, supervaluations) are needed then to capture the clear cases of presupposition: they follow from the boolean structure of the relevant categories.

4. ENRICHING THE TYPES

4.1. Since the types for most categories are boolean algebras it is not surprising that subcategories of a category may be defined according as the expressions in them must satisfy one or another boolean conditions on their denotations. For example, the APs *female, tall, skillful, fake,* and *alleged* all belong to logically distinct subcategories of AP, distinguished in terms of the boolean properties of the functions which can interpret them. (And each can be booleanly distinguished from the irrealis APs like *imaginary* if the semantics suggested in Section 1 is adoped.) And one of the ultimate aims of a semantic investigation of a language is to state the "meanings" of each expression in the language. While we are very far from that goal, being able to discriminate subclasses of expression which are grossly semantically similar is a positive step. So the fact that many expressions can be semantically distinguished in terms of the boolean properties of their denotations is a positive recommendation for a boolean approach to semantics.

Furthermore, most categories will be interpreted by functions from one boolean algebra to another, and hence distinct categories may be compared as to whether the conditions used to distinguish their subcategories are the same or not. And many striking similarities emerge. E.g. the logical subcategory features we need for APs overlap very significantly with those we need for adverbs, but almost not at all for those we need for VPs. VPs on the other hand are logically very similar in terms of subcategorization to TVPs, heads of possessives, etc. Grouping together categories which share many logical subcategorization features we find that they correspond reasonably well to natural syntactic classes (or super classes), which further supports the claim that the syntactic structure of a language reveals its logical structure.

Table I below is a first and very incomplete attempt to state these syntactic and logic correlations. On the left we give three syntactically defined classes of expressions and in the columns on the right which classes are subcategorized for which logical properties. The syntactic classes are: Modifiers, Predicatives, and Specifiers. *Modifiers* (Mods) are expressions which combine with elements of various categories to yield expressions in the same category. They will include APs, adverbs, PPs, ad-adjectives (e.g. *very* in *very tall*, etc.) and perhaps some ad-determiners (e.g. *very, too,* etc. in *very many, too few*).

Predicatives (Preds) are expressions which combine with full NPs and various "nominalized" structures, e.g. nominalized S's (including $\bar{S}$'s), VPs, etc. They include the VPs, TVPs, Ditransitive VPs, Prepositions, "transitive" CNPs e.g. *relative (of), employer (of),* etc. and heads of possessives (somewhat debatably). E.g. we analyze *('s) father* as something which combines with an NP such as *every man* to form an NP, *every man's father*. Preds are further distinguished from Mods in that only Preds may impose case on their (NP) arguments.

Specifiers (Specs) hardly constitute a super class since the only clear cases are the Determiners (Dets): *every, a, no, some but not all,* etc., though possibly the *to* in *to smoke is unhealthy* and possibly sentence complementizers will ultimately be included here as well. Semantically Specs map a set into a set of a higher type (extensionally, its power set).

TABLE I

Syntactic Class	Logical Subcategorization Features: restrict. absolute	preserve structure (Hom.)	increase decrease conservative	reverse polarity	transparent
Mods	yes	never	———	(?) no	yes
Preds	(never)	yes	-----	yes	yes
Specs	-----	never	yes	(?) yes	yes

I shall first discuss the interpretation of the entire Table and then present some of its entries in more detail, defining the logical subcategorization features.

Column 1 says that Mods are commonly subcategorized according as they are restricting or absolute. Preds never are, though the line below *never* emphasizes that most of the failure is definitional. *restricting,* etc. are features only defined on functions from an algebra into itself, and most Preds are not functions like that, though those that are, like heads of possessives, are never subcategorized as restricting, etc. Specs are never functions of the right sort.

In column 2 we see that Preds are commonly subcategorized according as they preserve the boolean structure of their domains, that is, are homomorphisms as defined earlier. Mods and Specs never are, though logically they could be.

Column 3 days that features like *increasing,* etc. only apply to functions of the sort that Specs are. Column 4, *polarity reversal* (and polarity preservation) logically applies to all three syntactic superclasses and, due to limited investigation on my part, all entries are tentative. Still it seems to me that Mods will never be so subcategorized, Preds very likely will be, and Specs I am unsure of. (Many of them do reverse polarity, but that normally follows from the independent constraint on the Spec, as most are logical constants, so it is not likely that we need subcategories here, that is, Specs which are not logical constants but which must be constrained to be interpreted by polarity reversal functions.)

And in column 5 we see that *transparency* may and does subcategorize all superclasses. Moreover, in distinction to the other features *transparency* is *not* defined with reference to the boolean structure of the types. I mention it because it interacts in interesting ways with the other features. E.g. while it is logically possible for Mods to be ± *restricting* and ± transparent in all combinations, note:

<u>UNIVERSAL GENERALIZATION 1</u>. All transparent modifiers are restricting.

Likely further study of the distribution of subcategory features within and across superclasses will reveal further constraints on the logical form of natural language which do not follow from the definitions of the logical features themselves. E.g. restricting slightly an observation due to Montague we may say that lexical VPs are always transparent (derived VPs like *be required to be a citizen* need not be).

Turning now to specifics, consider first the Mods. They are interpreted by functions from an algebra into itself (see LT 81 for a generalization of the notion of a modifier). We define:

<u>DEFINITION 6</u>. A function f in $F_{B/B}$ is *restricting* iff $f(b) \leq b$, all b in B.

The most common and most productively formed APs in a language will be restricting. Thus, mixing levels, *tall(man)* ≤ *man*, which is equivalent to saying that the set of individuals which have the *tall man* property is a subset, necessarily, of those with the *man* property. Similarly, both *female* and *skillful* are restricting, but *fake* and *alleged* are not. We may then distinguish a proper subcategory of AP, call it AP_{+r}, whose type will be the set of restricting functions from T_{CNP} into T_{CNP}. APs not in this category will not be required to be interpreted by restricting functions.

Note further that some Ad-adjectives will be restricting. E.g. *very(tall)* ≤ *tall* (i.e. for all properties p, (*very(tall)*)(p) ≤ *tall*(p).

Similarly, manner adverbs and many basic PPs (*in the garden*) will be restricting. E.g. the individuals who are working carefully are a subset of those who are working, those who are working in the garden are a subset of those who are working, etc. Some adverbs of course, e.g. *possibly, apparently, allegedly, ostensibly*, etc. are not restricting.

Moreover, the set of restricting functions from an algebra B into itself possesses a natural boolean structure, which I shall call a restricting algebra, as follows: for f and g restricting functions and b in B, $(f \wedge g)(b) =_{df} f(b) \wedge g(b)$; analogously for joins. And $(f')(b) = b \wedge (f(b))'$; $0(b) = 0$, and $1(b) = b$. The interesting property of a restricting algebra is the definition of complement (note the analogy to the factive predicates here). It says e.g. that *a (not diligent) student* is not simply an object (e.g. my pen) which fails to be a diligent student, it must be a student which fails to be a diligent student.

So the cross categorial generalizations of Table I can be extended regarding Mods as follows: languages may present modifiers of major categories, they may be subcategorized as restricting or not, and boolean combinations of restricting Mods will be interpreted as per the restricting algebra defined above.

And within the restricting Mods there are still quite general subcategory distinctions to make. Consider the logical differences between "absolute" APs like *male, female, mortal*, etc. and "merely restricting" ones like *tall* and *skillful*. The former do more than merely restrict the property they modify, they actually determine another property. Thus to say that Mary is a female lawyer is to say that Mary is both a lawyer and a female individual. But to say that Mary is a tall lawyer does not entail that Mary is a tall individual (e.g. suppose that lawyers are all short compared to individuals generally). So Mary is a tall lawyer only commits us to Mary's being tall relative to lawyers, not to individuals generally. More generally the following argument is valid, but ceases to be when *female* is replaced by *tall* or by *skillful*: Mary is a female lawyer and Mary is an artist; therefore Mary is a female artist.

To build these observations into our semantics in a general way, note that the property of being an individual (i.e. existing) is the unit property of the set of extensional properties P. E.g. it is U if P is taken as P(U). So we define:

<u>DEFINITION 7</u>. f in $F_{B/B}$ is *absolute* iff $f(b) = b \wedge f(1)$, all b in B.

surprising that subcategories of a category may be defined according as the expressions in them must satisfy one or another boolean conditions on their denotations. For example, the APs *female, tall, skillful, fake,* and *alleged* all belong to logically distinct subcategories of AP, distinguished in terms of the boolean properties of the functions which can interpret them. (And each can be booleanly distinguished from the irrealis APs like *imaginary* if the semantics suggested in Section 1 is adoped.) And one of the ultimate aims of a semantic investigation of a language is to state the "meanings" of each expression in the language. While we are very far from that goal, being able to discriminate subclasses of expression which are grossly semantically similar is a positive step. So the fact that many expressions can be semantically distinguished in terms of the boolean properties of their denotations is a positive recommendation for a boolean approach to semantics.

Furthermore, most categories will be interpreted by functions from one boolean algebra to another, and hence distinct categories may be compared as to whether the conditions used to distinguish their subcategories are the same or not. And many striking similarities emerge. E.g. the logical subcategory features we need for APs overlap very significantly with those we need for adverbs, but almost not at all for those we need for VPs. VPs on the other hand are logically very similar in terms of subcategorization to TVPs, heads of possessives, etc. Grouping together categories which share many logical subcategorization features we find that they correspond reasonably well to natural syntactic classes (or super classes), which further supports the claim that the syntactic structure of a language reveals its logical structure.

Table I below is a first and very incomplete attempt to state these syntactic and logic correlations. On the left we give three syntactically defined classes of expressions and in the columns on the right which classes are subcategorized for which logical properties. The syntactic classes are: Modifiers, Predicatives, and Specifiers. *Modifiers* (Mods) are expressions which combine with elements of various categories to yield expressions in the same category. They will include APs, adverbs, PPs, ad-adjectives (e.g. *very* in *very tall*, etc.) and perhaps some ad-determiners (e.g. *very, too,* etc. in *very many, too few*).

Predicatives (Preds) are expressions which combine with full NPs and various "nominalized" structures, e.g. nominalized S's (including $\bar{S}$'s), VPs, etc. They include the VPs, TVPs, Ditransitive VPs, Prepositions, "transitive" CNPs e.g. *relative (of), employer (of)*, etc. and heads of possessives (somewhat debatably). E.g. we analyze *('s) father* as something which combines with an NP such as *every man* to form an NP, *every man's father*. Preds are further distinguished from Mods in that only Preds may impose case on their (NP) arguments.

Specifiers (Specs) hardly constitute a super class since the only clear cases are the Determiners (Dets): *every, a, no, some but not all,* etc., though possibly the *to* in *to smoke is unhealthy* and possibly sentence complementizers will ultimately be included here as well. Semantically Specs map a set into a set of a higher type (extensionally, its power set).

TABLE I

Syntactic Class	Logical Subcategorization Features: restrict. absolute	preserve structure (Hom.)	increase decrease conservative	reverse polarity	transparent
Mods	yes	never	———	(?) no	yes
Preds	(never)	yes	-----	yes	yes
Specs	-----	never	yes	(?) yes	yes

I shall first discuss the interpretation of the entire Table and then present some of its entries in more detail, defining the logical subcategorization features.

Column 1 says that Mods are commonly subcategorized according as they are restricting or absolute. Preds never are, though the line below *never* emphasizes that most of the failure is definitional. *restricting*, etc. are features only defined on functions from an algebra into itself, and most Preds are not functions like that, though those that are, like heads of possessives, are never subcategorized as restricting, etc. Specs are never functions of the right sort.

In column 2 we see that Preds are commonly subcategorized according as they preserve the boolean structure of their domains, that is, are homomorphisms as defined earlier. Mods and Specs never are, though logically they could be.

Column 3 days that features like *increasing*, etc. only apply to functions of the sort that Specs are. Column 4, *polarity reversal* (and

This says directly that a female lawyer is a lawyer and a female existent (individual). Absolute functions are obviously restricting (since b meet anything ≤ b). Moreover, the reader may check that meets, joins and complements of absolute functions are absolute, so these functions actually form a subalgebra of the restricting algebra, one which is in fact isomorphic to B! (Note that two absolute functions differ iff their value at 1 is different; that can be anything we like, so there are as many absolute functions (but not more) than elements of B.)

Not surprisingly then we find that absolute APs behave syntactically much more like CNPs than do the merely restricting ones. E.g. they occur closer to the CNP than the others *a tall female lawyer* but ?? *a female tall lawyer*; they occur predicatively without CNP heads, e.g. *Mary is a female*, but **Mary is a tall*. Like CNPs but in distinction to merely restricting APs they do not naturally take degree modifiers or have comparative or superlative forms. Note that the existence of superlative forms for non-absolute restricting Mods is a natural expectation on this semantics. E.g. *tall tall lawyer* is generally a different property from *tall lawyer*, as the former requires being tall relative to tall lawyers whereas the latter only requires being tall relative to lawyers. However, it follows from the definition of absolute that *female female lawyer* is the same property as *female lawyer*. So iteration is logically pointless. Yet iteration seems to be the natural interpretation for the basic degree modifier *very*. E.g. as a first step in the semantics for *very* we might require that *very*(f)(b) maps b onto f(f(b)).

Notice now the existence of absolute VP mods, and a generalization apparent on boolean approaches but perhaps not on others. First note that, holding the point in time constant (12a) and (12b) are L-equivalent:

(12) a. John [is singing in the garden (at t_0)]
b. John [(is singing and is in the garden) (at t_0)].

So extensionally *to sing in the garden* and *to sing and be in the garden* are the same. Thus the *in the garden* function maps a VP like *sing* onto the meet of *sing* with the fixed VP *be in the garden*. And this intransitive use of *be* means essentially exist (at a point in time), and that is just the unit element of the (extensional) VP algebra, that is, that is the VP homomorphism which assigns all individuals value t. So semantically a stative locative PP (e.g. ones formed from *in, on, at, near, behind*, etc.) but not in general others is a function f mapping a VP interpretation h onto (h ∧ $f(1_{VP})$). So to characterize this subcategory it is enough to categorize them as VP Mods, +absolute. The general definition of absolute for Mods takes care of the rest.

Note that on a non-boolean approach it is not at all apparent that there are any logical similarities between PP semantics and (absolute) AP semantics. And this similarity is supported by similarity in syntactic behaviour. E.g. just as absolute but not merely restricting APs may function like CNPs in certain contexts (mentioned above) so PPs (together with the constant *be*) function like VPs, (*John is in the garden*, etc.) but those merely restricting adverbs do not so function, **John is carefully*.

4.2. Homomorphisms

We have already discussed (transparent) VPs like *be bald, be a linguist*, which behave homomorphically on their arguments (subjects). Similarly, many (transparent) TVPs behave homomorphically. E.g. *kiss(Mary and Sally)* is semantically the same VP as *kiss(Mary) and kiss(Sally)*, so *kiss* preserves meets, etc. Of course many TVPs are neither transparent nor homomorphisms (homs), e.g. *seek*, etc. (To look for the President or the Vice Pres. is not necessarily the same as looking for the Pres. or looking for the Vice Pres., though in fact there seems to be an ambiguity here similar to that mentioned for the factive predicates in (6)). There is however an interesting cooccurrence generalization here. Namely, while it is logically possible for a TVP to be ± tr(ansparent) and ± hom, and yield VPs which are any combination of these two features, in fact it seems that +tr, +hom TVPs always yield +tr, +hom VPs. E.g. the VP *kiss(Mary)* is clearly transparent and a homomorphism. I can find no apparent logical reason for this, but it seems similar to the generalization noted earlier that lexical VPs are usually transparent. So it seems that when an n-place Pred is transparent on its argument it forms n-1 place ones which are transparent on theirs (n > 0). Similarly perhaps for the feature +hom.

Note also that for the +tr +hom n-place Preds (n is the number of NPs they ultimately combine with to form a Sentence or 0-place Pred) we have a quite general syntax and a correspondingly general semantics: The 0-place Preds are the sentences, and their type is (extensionally) 2, a complete and atomic boolean algebra. The n+1 place Preds are, categorially, P_n/NP, and semantically their type is the homomorphisms from T_{NP}

into T_{P_n}, regarded as a boolean algebra itself, the operations being defined pointwise on the individuals.

And it seems that this general notion of an n-place Pred is linguistically enlightening. For example, Passive can be formulated (see KEENAN 1980 for details) as an operation which universally maps n+1 place preds onto n-place preds, with of course a uniform semantics - that in (13) will do as a first approximation.

(13) $$(\ldots(\text{pass}(p_{n+1}))(x_1)\ldots)(x_n) = \bigvee_y (\ldots(p_{n+1}(x_n))(x_2)\ldots(x_{n-1}))(y),$$

where the x_i and y range over individuals. So this view leads us to expect the existence of passives of intransitives, as in Turkish, Latin (*curritur* = being run), etc. of the category sentence! Similarly, Causative can be given a general definition as a class of operations deriving n+1 place Preds from n-place ones, etc.

Note also that it is not only the "verbals" among the Preds which may behave homomorphically. Some "transitive" CNPs will. E.g. if John has the property expressed by *friend of both Bill and Mary* then he must have the property expressed by *friend of Bill and friend of Mary*, and conversely. Moreover, noting that in LT the extensional type for CNP is isomorphic to that for VP, it will follow that the extensional type for the homomorphic "transitive" CNPs will be isomorphic that for the +hom TVPs, actually justifying our informal use of "transitive" here and giving to expect that e.g. "nominalizations" of TVPs will yield "transitive" CNPs, e.g. *destroy (the city) / destruction (of) the city*, etc. Much more could be said here.

So Preds are commonly subcategorized as +hom or -hom. (Some -hom VPs will be: *love each other, be the two students I know best, ate the whole cow (between them)*, etc. Note that subjects of such predicates formed with *and* will probably require a different *and* from the one we have been treating booleanly. Thus not all the *and*'s in *Both John and Mary and (also) Bill and Emily love each other* can be treated as intersections, otherwise we obtain as a reading that expressed by *John, Mary, Bill, and Emily all love each other*.

It is interesting to query here why Mods and Specs (≈ Dets) are never subcategorized as +hom. For the Mods the features we have used are largely incompatible with being a homomorphism. E.g. suppose that f is both restricting and a homomorphism. Since it is restricting we have that $f(p') \le p'$, and since a hom. $f(p') = (f(p))'$. So $(f(p))' \le p'$, so $p \le f(p)$. But since f is restricting, $f(p) \le p$. Thus $f(p) = p$, and since p was arbitrary we have that f is the identity function. So only one restricting function is a homomorphism.

And it is easy to show that no *negatively restricting* function ($f(p) \le p'$, all p), the interpretation for APs in the *fake*, etc. class, can be homomorphisms (since $f(1) = 0$). Similarly, the irrealis ones will not preserve the unit. But why aren't there other subcategories of AP, say the poorly understood class of *apparent, alleged, ostensible*, which behave homomorphically? And why do not Dets as a class (or as logical constants) preserve the boolean structure?

4.3. Determiners

In the simplest cases Dets are functions from B into P(B). We define:

DEFINITION 8. f in $F_{P(B)/B}$ is *increasing* iff for all p,q,r in B, if $p \in f(r)$ and $p \le q$ then q in f(r). f is *decreasing* iff if $p \le q$ and q in f(r) then p in f(r).

For example, without argument, Dets like *every, a, the, three, most, more, than half, uncountably many*, etc. are increasing. Negations of increasing ones are decreasing, e.g. *not a, not every*, etc. as well as *no* (= *not a*), *at most three, fewer than three*, etc. Dets like *all but three, some but not all, exactly three*, etc. are neither increasing nor decreasing. While the features *increasing* and *decreasing* have received the greatest attention in the literature on Dets (logically speaking) it is the property of being *conservative* defined below which actually seems to characterize the (one-place, transparent) Dets in English.

DEFINITION 9. f in $F_{P(B)/B}$ is *conservative* iff for all p,q in B, $p \in f(q)$, iff $(p \wedge q) \in f(q)$.

This definition may seem unintuitive and "mathematical" at first sight but in fact it is based on a sound intuition, one closely related to the Fregean compositionality condition. First consider that some simple Dets clearly meet it. Suppose that every student has a property p, and let s be the *student* property. Well, then clearly every student has both s and p, that is $(s \wedge p)$. And if every student has $(s \wedge p)$ then in particular every student has p. So $p \in \text{every}(s)$ iff $(p \wedge s) \in \text{every}(s)$. Other simple Dets are equally easily reasoned, as are more complex ones like *some but not all* and the italic portions of (14) below:

(14) a. *Every* student *but John* left.

b. *More* students signed *than teachers who didn't (sign)*.

We refer the reader to KEENAN & STAVI (to appear) for a thorough exposition of the treatment of Dets in English presented here.

The intuition behind the definition is this: if d is an English Det we expect the interpretation of e.g. d(*student*) to depend in a substantive way on which individuals have the *student* property, and to not depend on ones that do not. d(*student*) could not e.g. refer to the properties shared by all cats. And Definition 9 captures (perhaps not as perspicuously as it might) this intuition. For it follows from Definition 9 that the value of a Det f at a property p is determined by those properties q in f(p) which are $\leq$ p and thus ones which *only* individuals having p have. More explicitly:

THEOREM 2. Let $A = \{q \in f(p): q \leq p\}$. Then $f(p) = \{r: r \wedge p \in A\}$.

PROOF. Let $s \in f(p)$. By the definition of conservative, $(s \wedge p) \in f(p)$, and since $(s \wedge p) \leq p$ then $(s \wedge p) \in A$, so $s \in \{r: (r \wedge p) \in A\}$. Going the other way, let $s \in \{r: (r \wedge p) \in A\}$. So $(s \wedge p) \in A$. By definition of A, then $(s \wedge p)$ in f(p). Thus $f(p) = \{r: (r \wedge p) \in A\}$. □

This characterization of Dets is interesting for two additional reasons. First it clarifies the difference between the kinds of functions Dets are as opposed to homomorphisms. Since the value of a Det at p depends only on properties $\leq$ p, it follows that a Det has the smallest range of possible values at the 0 property, the next smallest range of possible values at the atoms, etc. and the greatest range at the unit property. Homomorphisms are not at all like that. They must for example map the unit onto the unit so they have no choice at the unit property (though as there are Dets which map the unit onto the unit this fact is not incompatible with being a Det).

And second, this characterization brings out a certain similarity between Dets and the restricting APs, clearly the most widespread and productively formed of the APs. Namely, a restricting AP f must map p onto some q $\leq$ p. And a Det(p) is a function of a set of q $\leq$ p. This seems a reasonably natural analogue of "higher type" restricting AP (though the closest analogue would be a function mapping p simply onto a set of properties $\leq$ p, and such functions are not Dets).

4.4. Polarity reversal

Drawing on a number of insightful observations of FAUCONNIER (1979) and LADUSAW (1979) we define, for B and D any boolean algebras:

DEFINITION 10. f in $F_{D/B}$ *reverses polarity* iff for all x,y in B, if $x \leq y$, then $f(y) \leq f(x)$; and f *preserves polarity* if $f(x) \leq f(y)$.

Although not described in boolean terms of polarity reversal, Fauconnier and Ladusaw have pointed out interesting correlations between the presence of negative polarity items and polarity reversal (pr) operators. Roughly, a sufficient (but not necessary) condition for negative polarity items to occur is that they be under the scope of a pr operator.

Notice that negation (boolean complement) is a pr operator from an algebra into itself. In fact there we have $x \leq y$ iff $y' \leq x'$. In this sense all categories have pr operators. But again negation is a logical constant, so this fact follows from its independently constrained interpretation. It is not clear that we need to subcategorize a category for such operators. The best candidates for such subcategorization will be sentential Preds, e.g. *implausible, doubtful*, etc. and in a slightly more restricted sense that I do not have the space to define, the negative factives like *strange, surprising*, etc. Among other Preds, possibly the TVP *suspect* and a few "transitive" APs like *suspicious (of), afraid (of)*, have readings on which they reverse polarity. Ladusaw (op cit) further cites the interesting case of the negative preposition *without*.

Among the other superclasses, I know of no convincing cases of pr Mods ("transitive" APs etc. are not Mods, they are Preds). Among the Dets there are many. *every* reverses polarity and the complements of most other (not of *every*) "basic" Dets (see KEENAN & STAVI (op cit) for the definition of basic Det) do, e.g. *not a*, etc. But again as these are logical constants it is not clear that we shall have to specify a set of non-logically determinate Dets which are constrained to be interpreted by pr functions. Analogous claims hold for certain "transitive" S Mods, e.g. *if* (on its ordinary truth functional definition). (It is easily shown that for propositions p and q, if $p \leq q$ then *if* q $\leq$ *if* p in the sense that for all propositions r, *if* q *then* r $\leq$ *if* p *then* r.)

So the advantage of our boolean characterization is that we can describe what a variety of elements in different categories have in common logically, namely they reverse polarity, and thus give a uniform statement to many (but not all) of the observations in FAUCONNIER (op cit) and LADUSAW (op cit). I might only note in conclusion here that not all "negative" items will reverse polarity. For example the complements of restricting functions are still restricting and do not reverse polarity. So e.g. the *(not diligent) students* are not necessarily a subset of the *(not diligent) young students,* even though the young students are necessarily a subset of the students; and this follows on our analysis.

Polarity preservation seems somewhat less interesting than polarity reversal, as there seems to be no correlated syntactic property such as triggering negative polarity items. And again while many operators preserve polarity, e.g. homomorphisms always do, it seems unlikely that we will have to constrain subcategories as preserving polarity independently of constraints on subcategories or constants which we need anyway.

4.5. Transparency

Non-rigorously we may say that a function f from X into Y is *transparent* if for all p,q in X and all possible worlds j, if the extension of p in j, ext(p,j), = ext(q,j) then ext(f(p),j) = ext(f(q),j). The "definition" assumes that X and Y are extensional, that is, that the *ext* function is defined on them (cross J). How extensions are defined depends a bit on the category, and of course many categories, i.e. that for TVPs like *seek, want, need,* etc., are not extensions. In general, if the type for C is a set of functions with domain J, then ext(f,j) is just f(j). The properly functional categories are extensional iff their functions are transparent.

As is clear from the above informal sketch, *transparency* is not a specifically boolean property. And part of our interest in it has already been mentioned. Namely certain generalizations concerning the distribution of transparency and other logical subcategorization features appear to impose constraints on the logical form of natural language.

Moreover, the vague claim that transparency and the other features are independent and interact in interesting ways is itself significant. In a certain sense, intensional logic is made up to distinguish transparent from non-transparent operators. And it is natural to wonder whether non-transparent operators (the transparent ones are all representable in an extensional logic) exhibit any interesting boolean behaviour. And they do.

For example, among the "merely restricting" Mods, almost all are non-

transparent (*tall*, and few other one dimensional physical object modifiers are the exception here). So e.g. if *doctor* and *lawyer* have the same extension in j, i.e. the doctors and the lawyers are the same individuals, it clearly does not follow that *skillful doctor* and *skillful lawyer* have the same extension, as a given individual might be a skillful doctor but an inept lawyer. But these APs are still restricting, e.g. the skillful doctors in j must be a subset of the doctors in j, all j, so *skillful doctor* ≤ *doctor*, where the type for CNPs in the intensional logic is $F_{P/J}$, taken as a boolean algebra defined pointwise on J.

Similarly we can expect that there will be non-transparent homomorphisms among the Preds, though no examples were given in LT (78). And an algebraic observation (due to Edit Doron, pc) tells us where to look. For suppose that h is a transparent homomorphism. Construct a non-transparent one as follows: Fix a particular k in J, and define f_h by $f_h(x)(j) = h(x)(k)$. In other words, the value of the new function at an argument has as its extension in any j the value of h at that argument in a fixed world k. So if we had a way of referring to possible worlds in our language we could construct such non-transparent homomorphisms. And many candidates suggest themselves.

Consider for example date names. Arguably they specify possible worlds (perhaps sets of them). So from a transparent homomorphism like *be a woman* we should be able to form a non-transparent one like *be a woman in 1972.* This latter Pred clearly seems to be a homomorphism. E.g. if The President and the Vice President were women in 1972 then The President was and so was the Vice President, etc., so the Pred preserves meets, etc. But the function also seems clearly to not be transparent. For example, in our world (1980) *the Prime Minister of Israel* and *Begin* have the same extension. But *be a woman in 1972* holds of the former but not the latter, so it is not transparent.

And this observation generalizes to large classes of subordinate clauses (pointed out to me by David Gil, pc) assumed here to be VP Mods, as is standard in generative grammar. Thus we obtain judgments similar to the ones above if *in 1972* is replaced by *when Nixon was President,* etc. *if* clauses behave similarly.

Yet another case of "possible world fixing" is illustrated by the "picture PP's" discussed in REINHART (1976). Thus from a transparent homomorphism like *cry* we may form a non-transparent one like *cry in Ben's picture*. Clearly, if the President and the Vice President are crying in

Ben's picture then the President etc. is, and conversely. So it preserves meets. And if no political figure is crying in Ben's picture then it is not the case that a political figure is crying in Ben's picture, so it appears to preserve complements. And equally it is not transparent. For if, say in our world, the President is the commander of the armed forces, we cannot infer from *the President is crying in Ben's picture* that a five starred general is, since Ben's picture may not have portrayed the one crying as a military figure at all but only as a civilian one.

Among other Preds, it seems likely that many transitive CNPs are non-transparent homs. Arguably if the property of being *onerous* is a member of *the duties of the President and the Vice President* then it is among *the duties of the President* and also among *the duties of the Vice President*, so arguably (but more work needed here) *duties (of)* preserves meets, etc. But it is clearly not transparent since if the President is the commander of the Army it will not follow that the duties of the President are the same as the duties of the commander of the Army, since *duties of* and many other such expressions pertain to roles or offices, not the individuals which hold them.

So we may infer here that non-transparent operators will also present an interesting boolean behaviour and that in particular Preds wil exhibit members in all combinations of subcategories ±transparent, ±homomorphism (the non-homomorphisms presented earlier are transparent).

5. CONCLUSION

Given that essentially all types are boolean it is not surprising that this boolean structure is used in natural language for reasons other than merely interpreting conjunctions, disjunctions, and negations. And Boole's suggestion that these operations represent "Laws of Thought" seems reasonable.

On the other hand we can expect that specific categories will present structure specific to what they describe, structure that is not specifically boolean. For example, an explicit semantics for Det will require features (or constants) defined in terms of cardinalities, properties which are not specifically boolean. Probably the merely restricting AP semantics will require discussions of "scalar" functions which are possibly not entirely definable in terms of the boolean ≤ relation (though we get far here). Doubtless the semantics for place and time adverbials and verbal subclasses will require analysis of our "natural geometry"; the semantic differences between verbs of motion, desire, and perception will doubtless require serious analysis of motion, desire, and perception, and there is no reason to think they are specifically boolean.

So a boolean approach to semantics is clearly not the whole story, but it is an important chapter.

FOOTNOTES

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1. Syntactically the analysis proposed here is similar to that developed in Delacruz' 'Factives and Proposition Level Constructions in Montague Grammar' (DELACRUZ 1976). Semantically there are major differences however. In particular I take the type for sentence complements to be the boolean closure of the I_p's, not simply the set of I_p's as in Delacruz. It is this which allows denotations for *everything that John believes*, etc. More importantly, I distinguish the subcategory of *strange* from that of *true, obvious*, etc. the two classes of adjective lying in different subalgebras of the set of homomorphisms from $T_{\bar{S}}$ into T_S, and for me it is this subcategory difference which accounts for the difference in presupposition. For Delacruz it is the interpretation of *the fact that S* which accounts for its presuppositional nature. Thanks to Susan Ben Chorin, Jeroen Groenendijk and Martin Stokhof for pointing out the similarity to Delacruz (op cit).
2. Note that as regards elements of P (as opposed to $F_{P/J}$, the type for CNP in a standard intensional logic) we can not normally distinguish extensional properties from others, *if* P is complete and atomic as is usual. But if P is not required to be atomic it will now be the case that there will be many, in fact infinitely many, properties in exactly one individual, no matter what one we chose. Only one of those however will be an atom (the one whose meet with the meet of the complements of all the atoms is the zero property). It is of course in the larger class of properties that we will take denotations of *imaginary horse*, etc. The extension of a property may be defined as its meet with the join of all the atoms, and a property will be called *extensional* iff it is ≤ to the

join of all the atoms. If we think of the join of the atoms as being the denotation for *real* then p is extensional iff $p = p \wedge real$. Clearly, the English expressions I cited as being naturally interpreted by atoms will meet this condition. That is, when we speak of the *tallest man* we are not normally comparing against Paul Bunyan, etc. So the fact that we cannot characterize atoms as the properties which are in exactly one individual (in a non-atomic algebra) does not argue that the notion of an atom is unclear pretheoretically. They are still the intended denotations of expressions like *tallest man, student who is the only student who passed,* etc.

3. Susan Ben Chorin provides me with the following simpler and direct proof: For x arbitrary in P, assume $x \leq \wedge I$. We must show that $x = 0$ or $x = \wedge I$, whence $\wedge I$ is an atom. From the definition of individual, $x \in I$ or $x' \in I$. Suppose $x \in I$. Then $\wedge I \leq x$, whence, from the assumption plus assymetry of $\leq$, $x = \wedge I$. Suppose $x' \in I$. Then $\wedge I \leq x'$, whence by transitivity of $\leq$, $x \leq x'$, so $x = x \wedge x' = 0$.

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